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Mohamed Boutahar, Jamel Jouini

GREQAM

Groupement de Recherche en Economie
Quantitative d'Aix-Marseille - UMR-CNRS 6579
Ecole des Hautes Etudes en Sciences Sociales
Universités d'Aix-Marseille II et III

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Mohamed BOUTAHAR*
Faculté des Sciences de Luminy
Université de la Méditerranée, France

Jamel JOUINI**
F.S.E.G.N., E.S.S.A.I. and L.E.G.I.
Université 7 Novembre de Carthage, Tunisie
GREQAM, Université de la Méditerranée, France

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Abstract: Some structural break techniques defined in the time and frequency domains are presented to explore, at the same time, the empirical evidence of the mean and covariance instability by uncovering regime-shifts in some inflation series. To that effect, we pursue a methodology that combines two approaches; the first is defined in the time domain and is designed to detect mean-shifts, and the second is defined in the frequency domain and is adopted to study the instability problem of the covariance function of the series. The proposed methodology has a double interest since, besides the detection of regime-shifts occasioned in the covariance structure of the series, it allows taking into account the presence of mean-shifts in this series. Note that unlike the works existing in the literature which often adopt a single technique to study the break identification problem, our methodology combines two approaches, parametric and nonparametric, to examine this problem.

Key-words: Structural change, mean and variance shifts, parametric and nonparametric approaches.

JEL Classification: C22, E31.

* Mohamed BOUTAHAR, Département de Mathématiques, case 90, Faculté des Sciences, 63 avenue de Luminy, 13288 Marseille cedex 9, France. Tel.: +33 4 91 82 90 89; fax: +33 4 91 82 93 56; E-mail: boutahar@univmed.fr.

** **Corresponding author:** Jamel JOUINI, Faculté des Sciences Economiques et de Gestion, Campus Universitaire Mrezgua, 8000 Nabeul, Tunisie. Tel.: +216 72 232 205; fax: +216 72 232 318; E-mail: Jamel.Jouini@fsegn.rnu.tn.

1 Introduction

The majority of works on structural change in the literature focus on the instability of the mean or the instability of the variance. For the first issue, we are referred to Yao (1987), and Yao and Au (1989) for the instability of the unconditional mean; Liu et al. (1997), and Bai and Perron (1998) for the instability of the conditional mean. For the second issue, two kinds of instability are also examined: i) the instability of the unconditional variance (the readers are referred to the works of Pagan and Schwert (1990), and Inclan and Tiao (1994) for tests developed in the time domain, and of Priestley (1965), Priestley and Rao (1969), Adak (1998), Ahamada and Boutahar (2002), and Mikosch and Starica (2004) for tests developed in the spectral domain); ii) the instability of the conditional variance (the readers are referred to Engle (1982), and Bollerslev (1986)).

The study of the instability in the time domain of the mean and the variance of a time series is fundamental in economics. Indeed, the theory often suggests that the economic factors consider the mean and the variance of the economic variable in their study. In the financial theory, the mean and the variance of the returns play an important part in portfolio management. In macroeconomics, Lucas (1973) affirms that the response of the inflation to varied chocks depends on his variance. The usefulness of the study of the instability of the moments of these economic or financial variables has been illustrated by several authors. We cite here two examples; the first concerns the mean instability of the inflation. Indeed, the works of Nelson and Plosser (1982), Fuhrer and Moore (1995), Stock (2001), among others, conclude that the inflation persistence,[1] noted ρ and often measured by the autocorrelation of order one in the autoregressive structure,[2] is very high, even equal to one. This indicates that the inflation is a random walk, which is an indicator of economic disequilibrium that is unacceptable by the central banks. However, the application of structural change tests on the inflation always leads to mean-shifts. Moreover, if we take into account these breaks in the mean, then the inflation persistence becomes reasonable (see, Bilke (2005) for the French inflation, and Levin and Pigger (2004) for the inflation of the industrialized countries). The second example treats the variance instability of financial data as considered by Mikosch and Starica (2004) who show the difficulty of describing long financial series with tools retaining the stationarity hypothesis of the covariance structure. In other words, a systematic adaptation of the parameters of the models used to describe long financial series must be done. Using series of large size, they explain some classic stylized facts (long-range dependence) by regime-shifts of the unconditional volatility. In the same context, Nouira et al. (2004) draw two characteristics a priori contradictory and yet coexistent in the daily returns of exchange rate euro/US dollar. They indeed show the non-stationarity of the covariance structure of the series and, after the extraction of the unstable variance using the algorithm based on the cumulative sums of squares of Inclan and Tiao (1994), the existence of long-memory in the filtered series.

In the literature, the works treating together the instability of the mean and the variance are almost nonexistent. To illustrate the limits of the above-mentioned works, we consider the following process:

[1] Note that the inflation persistence can be defined as the tendency of inflation to converge slowly (or sluggishly) towards the central bank's inflation objective, following changes in the objective or various other shocks. Documenting this property of inflation is important for a number of reasons such as its relevance for forecasting.

[2] In this context, Gadzinski (2005) provides results on the level of inflation persistence for the EU countries, the euro area and the US using six different inflation series. This is done by constructing two different measures of persistence on the basis of univariate models, namely the sum of the autoregressive coefficients and the half-life indicator; these two indicators can offer complementary information under certain circumstances.

$$Y_t = \mu_t + \sigma_t w_t, \qquad 1 \le t \le T, \tag{1}$$

where w_t is a stationary process of mean zero and variance σ_w^2.[3] The tests developed by Yao (1987), Yao and Au (1989), Liu et al. (1997), and Bai and Perron (1998) cannot detect the changes in variance of the process (1), $\mathrm{var}(Y_t) = \sigma_t^2 \sigma_w^2$, because in the structure of the process supposed by these authors, it is the change in $E(Y_t / I_{t-1}) = \mu_t$ that is examined, where $I_t = \sigma(Y_s, s \le t)$ is the sigma-algebra generated by the variables $(Y_s, s \le t)$. However, the break dates in the mean can be estimated using some methods robust to the autocorrelation and the heteroskedasticity.

The variance tests of Inclan and Tioa (1994), the tests based on the evolutionary spectral density of Priestley (1965), Ahamada and Boutahar (2002), and the ARCH tests of Engle (1982) suppose that the studied process has a mean zero (or equal to a constant). Consequently, these tests cannot also be used to detect the breaks in variance of the process (1), unless $\mu_t = \mu$ for $1 \le t \le T$. This example shows the necessity of a methodology that combines the two approaches, parametric and nonparametric.[4] It then consists in applying some parametric structural change techniques on the series in order to detect eventual mean-shifts. Once the number of mean-shifts and their locations are obtained, and based on the mean corrected series,[5] we apply a nonparametric approach to determine the number of breaks and their locations in the covariance structure of the series. This methodology can also be generalized to the study of the instability of conditional moments. For the change in the mean, we essentially use the selection procedure based on a test developed in Bai and Perron (1998), and some information criteria. For the change in the unconditional variance, we focus on the series having different instability forms without supposing any particular structure for them. Because the covariance function of a stationary process is the Fourier transform of the spectral density, one of the best approaches to investigate the stability of the covariance structure is to study the stability of the spectral density. This approach has been adopted by Von Sachs and Neumann (2000), and then by Ahamada and Boutahar (2002) to develop stationarity tests of the covariance structure. In this paper, we similarly proceed. Firstly, we estimate a time spectral density based on the theory of the evolutionary spectrum of Priestley (1965, 1996). Then, we estimate the number of breaks and their locations by examining the changes of the form of the spectral density. Note that this problem, which is based on the nonparametric approach, is tackled by boiling down to a parametric study of structural changes. Consequently, some parametric testing and estimation procedures can be applied to determine the number of breaks and their locations in the covariance structure of the

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[3] We can, for example, represent the mean and the variance by

$$\mu_t = \begin{cases} \mu_1 & \text{if} \quad 1 \le t \le T_1 \\ \mu_2 & \text{if} \quad T_1 < t \le T_3 \\ \mu_3 & \text{if} \quad T_3 < t \le T, \end{cases} \qquad \sigma_t = \begin{cases} \sigma_1 & \text{if} \quad 1 \le t \le T_2 \\ \sigma_2 & \text{if} \quad T_2 < t \le T, \end{cases}$$

where $T_1 < T_2 < T_3$, $\mu_i \ne \mu_{i+1}$ $(i = 1, 2)$ and $\sigma_1 \ne \sigma_2$.

[4] Unlike the parametric approach that requires the modelling of data series before their treatment, the nonparametric approach has the advantage to treat a large number of models since it does not require any particular structure.

[5] This means that we take into account the presence of mean-shifts in the considered series. The obtained series does not present, in principle, mean-shifts in its structure.

considered processes.[6] Thus, this methodology has a double interest. Indeed, in addition to the determination of regime-shifts occasioned in the covariance function of the series, it allows taking into account the presence of mean-shifts in the series.

The paper is organized as follows. Section 2 presents the parametric approach, namely the structural change model, the estimation method to estimate the regression coefficients and the break dates, and the selection procedures of the number of breaks. Section 3 defines the nonparametric approach. Indeed, we first briefly describe the spectrum theory and recall the Priestley's (1965, 1996) approach concerning the evolutionary spectral density theory and his estimator. We then show how the evolutionary spectral density can be used to locate the regime-shifts in the covariance structure of time series. Section 4 presents the methodology proposed to analyse the breaks in the mean and covariance structure of time series. Empirical applications based on inflation series[7] are provided in Section 5 to illustrate the usefulness of the proposed methodology. The results reveal some changes in the unconditional volatility of the considered data series. They also suggest that unlike the case of the study of the mean-break problem, the information criteria are more powerful than the sequential selection method in detecting the number of structural breaks in the covariance structure of the series based on the evolutionary spectrum theory. Section 6 concludes the paper. The results are provided in Appendix 1, and the different graphics of the series in Appendix 2.

2 Parametric Approach

For the study and the analysis of structural change models, the estimation of the number of breaks receives an important attention from the researchers. Several selection procedures exist in the literature, including the information criteria and the procedures based on a sequence of tests. These statistical tools are designed, in the context of this paper, to study the mean-shifts in the data series.

2.1 The structural change model and estimation method

Consider the following mean-shift model with *m* breaks:

$$Y_t = \mu_j + X_t, \qquad t = T_{j-1}+1,\ldots,T_j, \tag{2}$$

for $j = 1,2,\ldots,m+1$, $T_0 = 0$ and $T_{m+1} = T$. Y_t is the observed dependent variable (the inflation series in this paper), μ_j are the means with $\mu_i \neq \mu_{i+1}$ $(1 \leq i \leq m)$, and X_t is the error term. The break dates $(T_1,\ldots,T_m)$ are explicitly treated as unknown. Let $\mu = (\mu_1, \mu_2, \ldots, \mu_{m+1})'$. The estimation method proposed in Bai and Perron (1998) is based on the ordinary least-squares (OLS) principle. It first consists in estimating the regression coefficients μ_j by minimizing the sum of squared residuals $\sum_{i=1}^{m+1}\sum_{t=T_{i-1}+1}^{T_i}(Y_t - \mu_i)^2$. Once the estimate $\hat{\mu}(T_1,\ldots,T_m)$ is obtained, we substitute it in the objective function and denote the resulting sum of squared residuals as $S_T(T_1,\ldots,T_m)$. The estimated break dates $(\hat{T}_1,\ldots,\hat{T}_m)$ are then determined by

[6] Note that our approach is original since unlike the works existing in the literature, which often adopt a single technique to study the identification problem of structural breaks, it combines the two techniques, parametric and nonparametric, to examine this problem.

[7] We study the inflation because it is of fundamental importance since this variable can have far-reaching implications for the economy both in terms of economic efficiency and wealth distribution.

minimizing $S_T(T_1,...,T_m)$ over all partitions $(T_1,...,T_m)$ such that $T_i - T_{i-1} \geq [\varepsilon T]$,[8] where ε is an arbitrary small positive number and [.] denotes the integer part of argument. Finally, the estimated regression coefficients are such that $\hat{\mu} = \hat{\mu}(\hat{T}_1,...,\hat{T}_m)$. In our empirical computations, we use the efficient algorithm developed in Bai and Perron (2003), based on the principle of dynamic programming, to estimate the unknown parameters.

2.2 Selection of the number of breaks

To select the number of breaks and their locations, Yao and Au (1989) propose the criterion

$$YIC(m) = \ln\left(S_T\left(\hat{T}_1,...,\hat{T}_m\right)/T\right) + mC_T/T, \tag{3}$$

where $p^* = 2m+1$ is the number of unknown parameters, and $\{C_T\}$ is any sequence satisfying $C_T T^{-2/n} \to \infty$ and $C_T/T \to 0$ as $T \to \infty$ for some positive integer *n*. In our empirical applications, we use the sequence $C_T = 0.368T^{0.7}$ proposed by Liu et al. (1997) who suggest the following modified Schwarz' criterion:

$$MIC(m) = \ln\left(S_T\left(\hat{T}_1,...,\hat{T}_m\right)/\left(T - p^*\right)\right) + 0.299p^*[\ln(T)]^{2.1}/T. \tag{4}$$

The estimated number of break points *m* is obtained by minimizing these criteria given an upper bound *M* for *m*.[9] Bai and Perron (1998) suggest a method based on the sequential application of the following statistic:[10]

$$\sup F_T(l+1 \mid l) = \left\{S_T\left(\hat{T}_1,...,\hat{T}_l\right) - \min_{1\leq i\leq l+1} \inf_{\tau\in\Lambda_{i,\varepsilon}} S_T\left(\hat{T}_1,...,\hat{T}_{i-1},\tau,\hat{T}_i,...,\hat{T}_l\right)\right\}/\hat{\sigma}^2, \tag{5}$$

where $\Lambda_{i,\varepsilon} = \left\{\tau;\ \hat{T}_{i-1} + \left(\hat{T}_i - \hat{T}_{i-1}\right)\varepsilon \leq \tau \leq \hat{T}_i - \left(\hat{T}_i - \hat{T}_{i-1}\right)\varepsilon\right\}$, $S_T\left(\hat{T}_1,...,\hat{T}_{i-1},\tau,\hat{T}_i,...,\hat{T}_l\right)$ is the sum of squared residuals resulting from the least-squares estimation from each *m*-partition $(T_1,...,T_m)$, and $\hat{\sigma}^2$ is a consistent estimator of σ^2 under the null hypothesis.[11] The procedure to estimate the number of breaks is the following:

- Start by estimating a model with small number of break dates (or with no break) using the global minimization of the sum of squared residuals (section 2.1) or the sequential method one-at-a-time proposed by Bai (1997).

[8] From Bai and Perron (2003), if the estimation is the sole concern for the study, then the minimal number of observations in each regime $[\varepsilon T]$ can be set to any value greater than 1, the number of regressors in the model.

[9] The performance of these criteria has been recently examined by Boutahar and Jouini (2007) who consider the problem of selecting the number of breaks in the mean of a time series. Indeed, they prove analytically and show by a Monte Carlo study that the above criteria tend to choose a spuriously high number of structural breaks when the process is trend-stationary without changes. The important question suggested by their results is that of distinction between trend-stationary process and random walk when modelling real data series.

[10] This statistic allows testing the null hypothesis of *l* breaks against the alternative that an additional break exists.

[11] Note that the asymptotic critical values relating to this test are provided in Bai and Perron (1998, 2003) for some values of the trimming ε and the maximum possible number of breaks *M*.

- Perform parameter constancy tests for each subsample (those obtained by cutting off at the estimated break points), adding a break to a subsample associated with a rejection with the test $\sup F_T(l+1|l)$.
- Repeat the process by increasing l sequentially until the test $\sup F_T(l+1|l)$ fails to reject the no additional structural change hypothesis.

The final number of breaks is thus equal to the number of rejections obtained with the parameter constancy tests plus the number of changes used in the initial step. Note that, unlike the information criteria, this procedure can directly take into account the effect of possible serial correlation in the errors and heterogeneous variances across regimes. Bai and Perron (2003, 2006) favour the sequential method based on the $\sup F_T(l+1|l)$ test which seems to perform better than procedures based on information criteria.

Jouini and Boutahar (2005) use the above-mentioned selection methods to explore the empirical evidence of the instability by uncovering structural breaks in some U.S. time series. To that effect, they pursue a methodology composed of different steps and propose a modelling strategy to implement it. Their results indicate that the time series relations have been altered by various important facts and international economic events such as the two Oil-Price Shocks and changes in the International Monetary System.

3 Nonparametric Approach

In this section, we present a nonparametric approach based on the evolutionary spectrum theory of Priestley (1965, 1996) allowing the detection of breaks in the covariance structure of time series. It seems that some stationarity tests defined in the time domain, like those proposed by Pagan and Schwert (1990), and Kwiatkowski et al. (1992), are specific to particular forms of nonstationarity.[12] An efficient way, to capture more instability forms in the covariance structure of time series, is the use of the spectral density since this latter is linked to the covariance function through the Fourier transform. The interest of the evolutionary spectral density is great since it incidentally allows modelling the series, and locating the instable frequencies and the dates from which these instabilities happened. So, the evolutionary spectral density has this unique particularity to provide simultaneously the instability characteristics of the series in the time and frequency domains. We can then check, in the case of nonstationarity, the frequential components of the instability. Indeed, if the frequencies are low, then the instability concerns the long term. However, if the frequencies are high, then the instability affects the short term.

3.1 Theory of the evolutionary spectrum

The nonstationary processes in the spectral domain have been studied by several authors in the literature such as Priestley (1965), Priestley and Rao (1969), Adak (1998), among others. Here, we borrow from Priestley (1965) the ideas and the notations.[13] The spectral analysis constitutes an approach of the analysis of processes analogous to the one of that of the time domain which is based on the autocovariance function. It brings additional informations for

[12] The approach of Pagan and Schwert (1990) is essentially based on the homogeneity of the variance of the studied process. This can be insufficient to detect the nonstationarity of processes having a variance approximately constant and a covariance structure dependent of time. The KPSS test (Kwiatkowski et al., 1992) is often evoked, but it is principally centered on the nonstationarity explained by the presence of unit-root.

[13] For the details and proofs, the readers are referred to this work.

the interpretation of some results and allows avoiding the introduction of a parametric model that is not always ad hoc.

3.1.1 Definition

The theory of the evolutionary spectrum of Priestley (1965) is concerned with oscillatory processes $\{X_t\}$ defined as follows:

$$X_t = \int_{-\pi}^{\pi} A_t(\omega)\, e^{i\omega t}\, dZ(\omega), \tag{6}$$

where for each ω, the sequence $\{A_t(\omega)\}$, as function of *t*, has a generalized Fourier transform whose modulus has an absolute maximum at the origin. $\{Z(\omega)\}$ is an orthogonal process on $[-\pi,\pi]$ with $E[dZ(\omega)] = 0$, $E\left[\,|dZ(\omega)|^2\,\right] = d\mu(\omega)$, where $\mu(\omega)$ is a positive measure. Unlike the stationary case where the spectral representation provides a frequential description of the process, the oscillatory processes have an evolutionary spectrum that provides a frequential description which is local and at each date t at the same time.[14] Without loss of generality, the evolutionary spectral density of the process $\{X_t\}$ is given by

$$h_t(\omega) = \frac{dH_t(\omega)}{d\omega}, \qquad -\pi \le \omega \le \pi, \tag{7}$$

where $dH_t(\omega) = |\,A_t(\omega)|^2\, d\mu(\omega)$. The Priestley's evolutionary spectrum theory is particularly an attractive concept since it has a physical interpretation. It encompasses most other approaches as special cases and includes many types of nonstationary processes. The instantaneous variance of $\{X_t\}$ is given by

$$\sigma_t^2 = \mathrm{var}(X_t) = \int_{-\pi}^{\pi} h_t(\omega)\, d\omega. \tag{8}$$

These relations show that all modifications in the time of the covariance structure of the studied series may be captured by studying the stability of the evolutionary spectral density $h_t(\omega)$. In particular, the relation (8) shows that a modification of the variance of the process necessarily entails a variation of $h_t(\omega)$ with respect to the variable time.

3.1.2 Estimation of the evolutionary spectral density

The proposed method is inspired by that of the stationary case with the constraint of local stationarity at the date *t*. An estimator of $h_t(\omega)$ at time *t* and frequency ω can be obtained using two windows $\{g_u\}$ and $\{w_v\}$. Without loss of generality, the estimator $\hat{h}_t(\omega)$ is constructed as follows:

[14] The description is local since it exists an interval of the frequency domain on which the process $\{X_t\}$ can be considered as approximately stationary.

$$\hat{h}_t(\omega) = \sum_{v \in Z} w_v \left| U_{t-v}(\omega) \right|^2 , \tag{9}$$

where $U_t(\omega) = \sum_{u \in Z} g_u X_{t-u} e^{-i\omega(t-u)}$. We choose the following windows $\{g_u\}$ and $\{w_v\}$:

$$g_u = \begin{cases} 1/\left(2\sqrt{k\pi}\right) & if \quad |u| \le k \\ 0 & if \quad |u| > k \end{cases} \quad \text{and} \quad w_v = \begin{cases} 1/T', & if \quad |v| \le T'/2, \\ 0, & if \quad |v| > T'/2. \end{cases} \tag{10}$$

From Priestley (1988), $E\left(\hat{h}_t(\omega)\right) \approx h_t(\omega)$, $\mathrm{var}\left(\hat{h}_t(\omega)\right)$ decreases as T' increases and $\forall\,(t_1, t_2)$, $\forall\,(\omega_1, \omega_2)$, $\mathrm{cov}\left[\hat{h}_{t_1}(\omega_1), \hat{h}_{t_2}(\omega_2)\right] \approx 0$ if at least one of the following conditions *(i)* or *(ii)* is satisfied:[15]

$$(i) \quad |t_1 - t_2| \ge T', \qquad (ii) \quad |\omega_1 \pm \omega_2| \ge \frac{\pi}{k}. \tag{11}$$

3.2 Detection of breaks in the covariance structure

Let $\{X_t\}_{t=1,\dots,T}$ be data from a discrete process $\{X_t\}$ with theoretical evolutionary spectral density $h_t(\omega)$. We consider the grid of times $\{t_i = T'i\}_{i=1}^{I}$, where $I = [T/T']$ and the grid of frequencies $\left\{\omega_j = \frac{\pi}{20}(1+3(j-1))\right\}_{j=1}^{k}$. This implies that $\{t_i\}$ and $\{\omega_j\}$ satisfy the above-mentioned conditions *(i)* and *(ii)*. Let $Z_{ij} = \ln\left(\hat{h}_{t_i}(\omega_j)\right)$, and $h_{ij} = \ln\left(h_{t_i}(\omega_j)\right)$. From Priestley and Rao (1969), we have

$$Z_{ij} \approx h_{ij} + e_{ij}, \tag{12}$$

where the sequence $\{e_{ij}\}$ is approximately uncorrelated and identically distributed normal. The model (12) may also be written as

$$Z_{i.} \approx h_{i.} + e_{i.}, \tag{13}$$

where $Z_{i.} = \frac{1}{k}\sum_{j=1}^{k} Z_{ij}$ and $h_{i.} = \frac{1}{k}\sum_{j=1}^{k} h_{ij}$. In other words, $Z_{i.}$ and $h_{i.}$ are respectively the means of logarithm of the estimated and theoretical spectral densities on the grid of frequencies $\left\{\omega_j = \frac{\pi}{20}(1+3(j-1))\right\}_{j=1}^{k}$. The number of taken values $h_{i.}$ depends on the number of regime-shifts of the series X_t. Indeed, on each interval where the series is

[15] For more details on the relations *(i)* and *(ii)* and the choice of *k* and T', the readers are referred to Priestley (1969).

stationary, the evolutionary spectral density is independent of time, i.e. $h_{i.}$ is constant with respect to i on each of the intervals corresponding to regime-shifts. The model is then with change in mean (mean-shift model). More precisely, if the series is stationary on a sequence of $m+1$ successive intervals $\{I_l\}_{l=1}^{m+1}$, with $I_l \subset \{t_i = T'i\}_{i=1}^{I}$, $I_l \cap I_{l'} = \emptyset$ if $l \neq l'$, and $\cup_{l=1}^{m+1} \{I_l\} = \{t_i = T'i\}_{i=1}^{I}$, then the model (13) becomes a mean-shift model with m breaks for the value of the spectral density $h_{i.}$, i.e.

$$Z_{i.} \approx h_l + e_{i.}, \tag{14}$$

where h_l is a constant,[16] $h_l = h_{i.}$ for all $t_i \in I_l$. In this paper, we consider that $T' = 1$, which means that we constrain the frequential resolution by supposing only the condition *(ii)* of (11) with $k = 7$.[17] This implies that the dates corresponding to structural changes will be chosen among the points of the grid $\{t_i = i\}_{i=1}^{T}$. Thus, we can apply the parametric testing and estimation procedures discussed above on the mean-shift model given by (14) to determine the number of breaks and their locations in the covariance structure of the considered processes.

Note that Ben Aïssa et al. (2004) adopt this nonparametric approach to propose a test similar to that based on Kolmogorov-Smirnov statistic applied to the evolutionary spectrum to determine the number of changes and their locations in the monthly U.S. inflation series.

Most of the works existing in the literature treat the instability question in the time by adopting parametric procedures. However, the numerical examples considered in this paper are designed to evaluate the performance of the above procedures by adopting a nonparametric approach. Note that the use of large size data is necessary for the efficiency of the implementation of this approach.

4 Methodology

The examination of some data series can often allow us to observe that it is difficult to accept that the mean and/or the variance of such processes are constant over the entire sample. Since the nonparametric approach based on the evolutionary spectral theory requires that the series is mean-stationary, we propose in this paper a methodology that combines the two adopted approaches; the first allows studying the instability question of the mean, and the second allows investigating the instability question of the covariance structure of time series. More exactly, this methodology is composed of two steps:

1. We use the parametric procedures robust to the autocorrelation and the heteroskedasticity (see, section 2) to estimate the break dates in the mean of Y_t, namely $\hat{T}_1, \hat{T}_2, \ldots, \hat{T}_{\hat{m}_1}$, where $\hat{m}_1$ is the number of mean-shifts in the series, based on the model (2).
2. We consider the mean corrected series $X_t = Y_t - \hat{\mu}_i$, for $t = \hat{T}_{i-1}+1, \ldots, \hat{T}_i$, where $i = 1, 2, \ldots, \hat{m}_1 + 1$, $\hat{T}_0 = 0$ and $\hat{T}_{\hat{m}_1+1} = T$, and $\hat{\mu}_i$ is a consistent estimator of μ_i. Then, we apply the nonparametric techniques on the series X_t to estimate the break dates in

[16] This is because the spectral density is independent of time in each interval where the process is stationary.
[17] For more details on the choice of T' and k, see Priestley (1965).

the covariance structure of Y_t, namely $\widetilde{T}_1, \widetilde{T}_2, \ldots, \widetilde{T}_{\hat{m}_2}$, where $\hat{m}_2$ is the number of variance-shifts in the series. Note that this is done by using the defined parametric procedures since the study of the covariance instability is in turn tackled by boiling down to a parametric study of structural changes.

5 Empirical illustrations

We now apply the methodology proposed above on the inflation series obtained using the Consumer Price Index (CPI) of four countries: France, Italy, Germany and U.S.A.. We consider monthly data covering the period 1957 : 2-2002 : 3 (542 observations) and obtained from the International Monetary Fund (IMF) database. A look at the graphs of the series (see, Appendix 2) allows us to observe that the mean and variance stationarity does not seem acquired since it is difficult to accept that these two moments are considered as constant over all the series. We start by applying the first step of the methodology that consists in detecting the mean-shifts of the series using the above-mentioned parametric procedures, namely the information criteria and the sequential selection procedure, based on the model (2). We suppose that the errors are correlated since we don't consider the case where a lagged dependent variable is allowed as regressor (see, Assumption A4 in Bai and Perron, 1998). We also consider different distributions for the regressors and the errors[18] across regimes since the graphs of the series display some variability in different periods. The maximum permitted number of breaks is $M = 5$, the minimum number of observations in each regime is $[\varepsilon T]$ where $\varepsilon = 0.10$, and the significance level for the sequential selection procedure is $\alpha = 5\%$.

The results provided in Tables 1 and 2 indicate that the information criteria detect two break dates for the first three countries and three dates for U.S.A., while the sequential method chooses an additional break date for both France[19] and Italy.[20] The detection of an additional break date by the sequential procedure indicates that the French and Italian inflation processes are unstable at the 1990s. These results contradict those of the information criteria that indicate that the evolution curves of French and Italian inflation series were flattened during almost the last twenty years making the processes stable. Consequently, this calls the attention to the fact that the used procedure is more powerful than the information criteria in detecting mean-shifts,[21] which confirms the conclusions of Bai and Perron (2003, 2006). The majority of the 95% confidence intervals[22] of the break dates cover small periods indicating that the dates are precisely estimated. A feature of substantial importance is that these dates are associated with high magnitude of change while the others are not. Thus, the estimation precision of the break dates highly depends on the break size of the dates.

After determining the mean-shifts by adopting the parametric procedures, we now focus on the nonparametric approach based on the evolutionary spectrum theory to detect the regime-shifts in the covariance structure of the inflation series. To that effect, we pursue the second

[18] Note that the existence of breaks in the variance could be exploited to increase the precision of the break date estimates (Bai and Perron, 2003).

[19] Note that the break date $\hat{T}_2 = 1985 : 5$ selected by the sequential procedure has also been detected by Bilke (2005). Indeed, by studying the same inflation series based on the CPI, over a shorter period (1973-2003), he only detects one break date. However, by restricting to the service sector, he detects an additional break date located in 1993 : 2.

[20] A look at the graphs of the series may confirm the selection of the additional dates as mean-shift points since the series can be affected by structural breaks as there is an anomalous behaviour at the 1990s.

[21] Note that the breaks in mean detected by the sequential procedure are illustrated on the graphs of the series given in Appendix 2.

[22] The confidence intervals are constructed based on the asymptotic distribution of the estimated break dates given in Bai and Perron (1998).

step of the proposed methodology. We recall that the mean corrected series X_t (see, Appendix 3) is obtained based on the breaks detected by the sequential method since it is more powerful than the information criteria, and this because it allows selecting dates even with a very small magnitude of change.[23] The application of the break selection procedures on these series does not provide any mean-shift, which allows concluding that the series are stable in mean,[24] and consequently we can use the nonparametric approach that requires the stability of the moment of order one before examining the instability question of the covariance structure of time series. The results presented in Table 3 indicate that unlike the conclusions deduced during the study of the mean-shift problem, the sequential selection procedure is not more powerful than the information criteria for the detection of the number of breaks and their locations in the covariance structure of the series based on the evolutionary spectrum theory.[25] This is explained by the fact that the sequential method underestimates the number of breaks in the covariance structure of the series since a look at the graphs of the mean corrected series (see, Appendix 3) allows observing that there is some breaks. The confidence intervals of the break dates indicate that some dates are precisely estimated. The results show that the covariance structure of the inflation series varies between some dates. The unconditional volatility of the series is not constant over these intervals, which confirms our initial intuition.

It seems that the obtained results confirm the recent works of some authors as Loretan and Phillips (1994), and Mikosch and Starica (2004).

6 Conclusion

In this paper, the problem of the mean and covariance instability has been subjected to a meticulous examination based on some techniques developed in the time and frequency domains. Indeed, we have proposed a methodology for studying, at the same time, the changes in the mean and covariance structure of time series. We have illustrated the usefulness of the methodology through a few empirical applications to inflation series. The obtained results highlight the practical importance of the proposed methodology. This paper is then justified by our aim to find a methodology that allows investigating the instability problem of the mean and covariance at the same time.

[23] Note that $\hat{m}_1$ is equal to 2 for Germany and 3 for the other countries.

[24] This can be corroborated by a look at the graphs of the series given in Appendix 3.

[25] Note that the breaks in variance detected by the information criteria are illustrated on the graphs of the mean corrected series reported in Appendix 3.

Appendix 1: Empirical results

Table 1. Estimate results for the mean-shift detection by the information criteria

Estimated break dates[1]				Estimated coefficients[2]			
France	$\hat{T}_1$	$\hat{T}_2$		$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\mu}_3$	
	1973:3	1984:10		0.0042	0.0086	0.0019	
	(72:4-76:2)	(84:7-85:4)		(0.0005)	(0.0004)	(0.0002)	
Italy	$\hat{T}_1$	$\hat{T}_2$		$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\mu}_3$	
	1972:6	1984:2		0.0027	0.0125	0.0037	
	(71:6-72:8)	(83:11-85:3)		(0.0003)	(0.0008)	(0.0003)	
Germany	$\hat{T}_1$	$\hat{T}_2$		$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\mu}_3$	
	1969:11	1982:6		0.0018	0.0043	0.0017	
	(66:10-73:5)	(80:7-85:4)		(0.0004)	(0.0004)	(0.0002)	
U.S.A.	$\hat{T}_1$	$\hat{T}_2$	$\hat{T}_3$	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\mu}_3$	$\hat{\mu}_4$
	1966:1	1973:1	1981:9	0.0013	0.0035	0.0075	0.0026
	(65:1-66:10)	(71:8-73:3)	(81:2-82:8)	(0.0002)	(0.0002)	(0.0005)	(0.0002)

Notes: [1] In parentheses are reported the 95% confidence intervals for the break dates.
[2] In parentheses are reported the standard errors (robust to serial correlation) for the estimated regression coefficients.

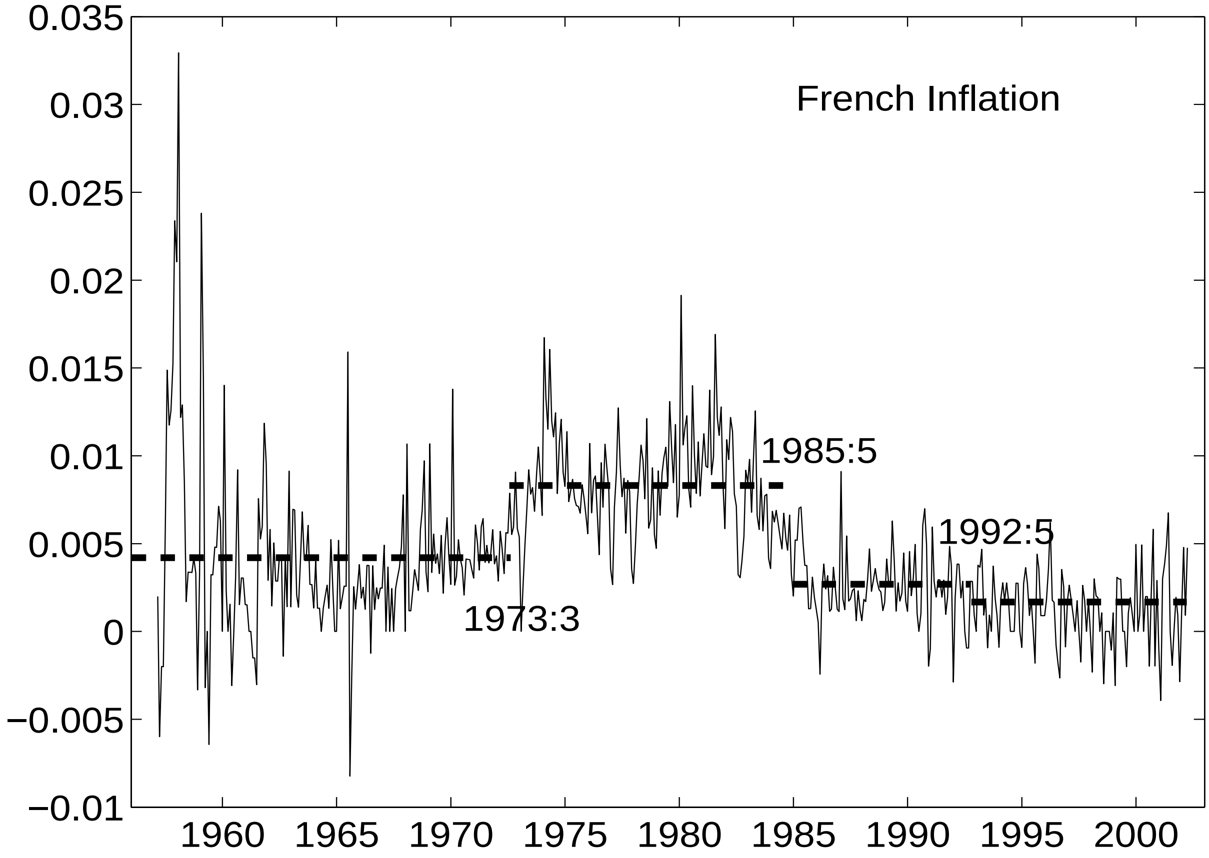
Table 2. Estimate results for the mean-shift detection by the sequential procedure

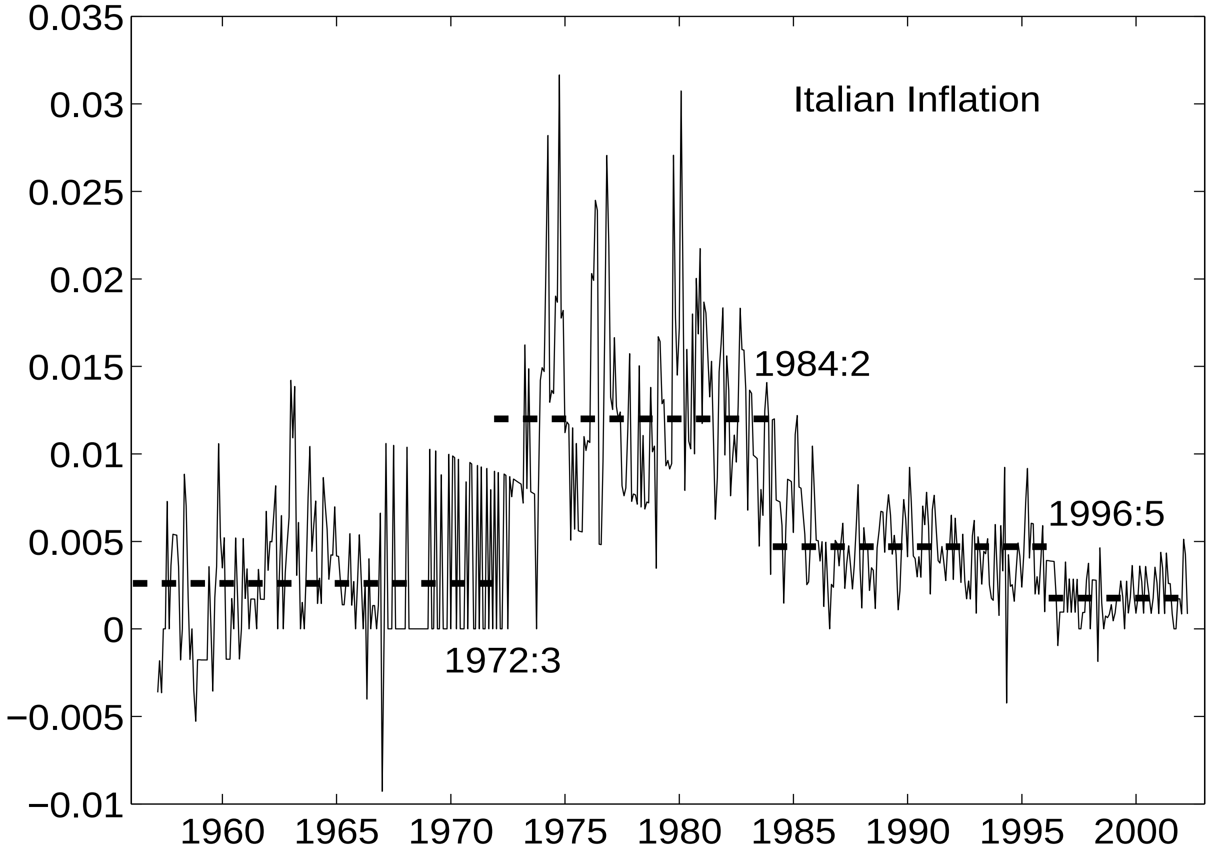
Estimated break dates				Estimated coefficients			
France	$\hat{T}_1$	$\hat{T}_2$	$\hat{T}_3$	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\mu}_3$	$\hat{\mu}_4$
	1973:3	1985:5	1992:5	0.0042	0.0084	0.0025	0.0012
	(72:2-76:5)	(85:3-86:1)	(89:1-94:9)	(0.0005)	(0.0004)	(0.0002)	(0.0002)
Italy	$\hat{T}_1$	$\hat{T}_2$	$\hat{T}_3$	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\mu}_3$	$\hat{\mu}_4$
	1972:3	1984:2	1996:5	0.0027	0.0124	0.0046	0.0018
	(71:3-72:5)	(83:12-85:7)	(96:2-97:8)	(0.0003)	(0.0008)	(0.0003)	(0.0002)
Germany	$\hat{T}_1$	$\hat{T}_2$		$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\mu}_3$	
	1969:11	1983:1		0.0018	0.0042	0.0017	
	(66:9-73:8)	(80:12-85:11)		(0.0004)	(0.0003)	(0.0002)	
U.S.A.	$\hat{T}_1$	$\hat{T}_2$	$\hat{T}_3$	$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\mu}_3$	$\hat{\mu}_4$
	1967:4	1973:1	1982:7	0.0015	0.0037	0.0073	0.0026
	(66:7-68:2)	(70:11-73:3)	(82:1-83:9)	(0.0002)	(0.0002)	(0.0005)	(0.0002)

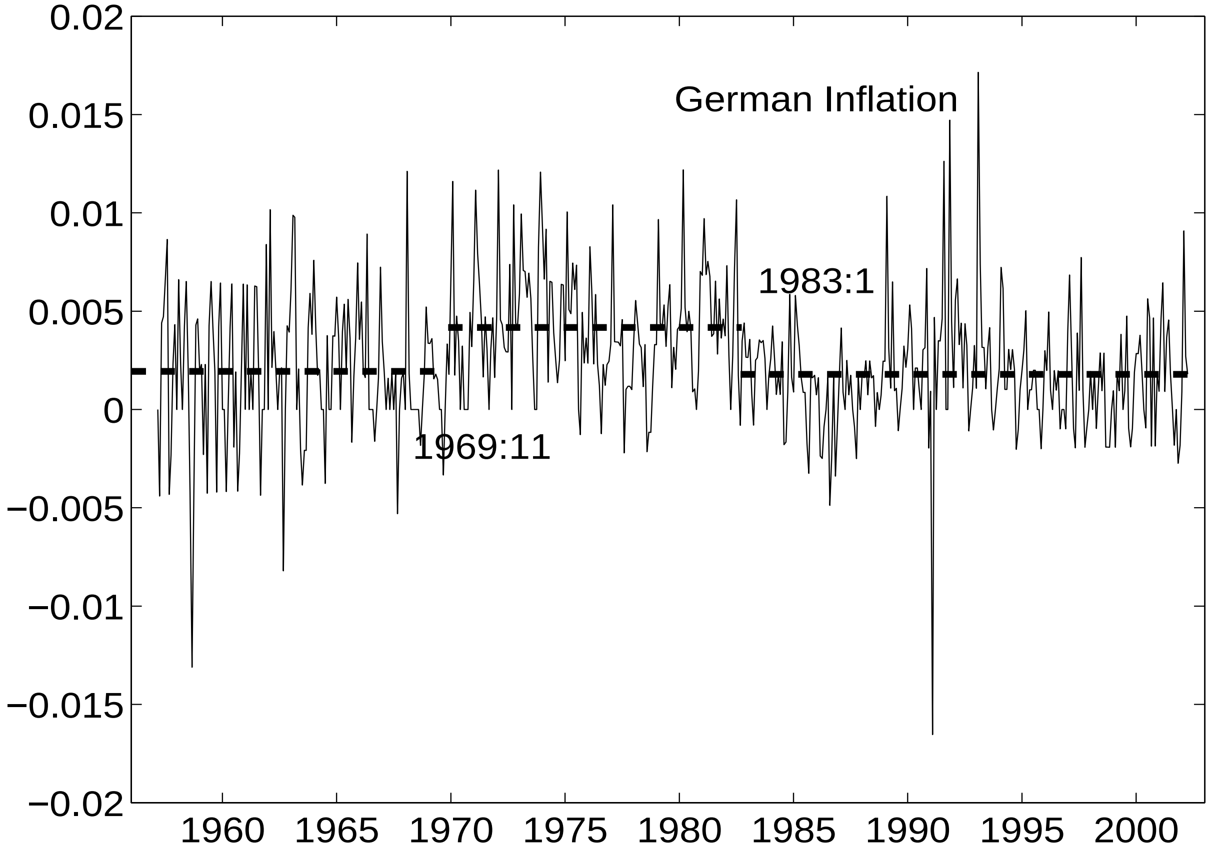
Table 3. Estimate results for the covariance-shift detection by the selection procedures

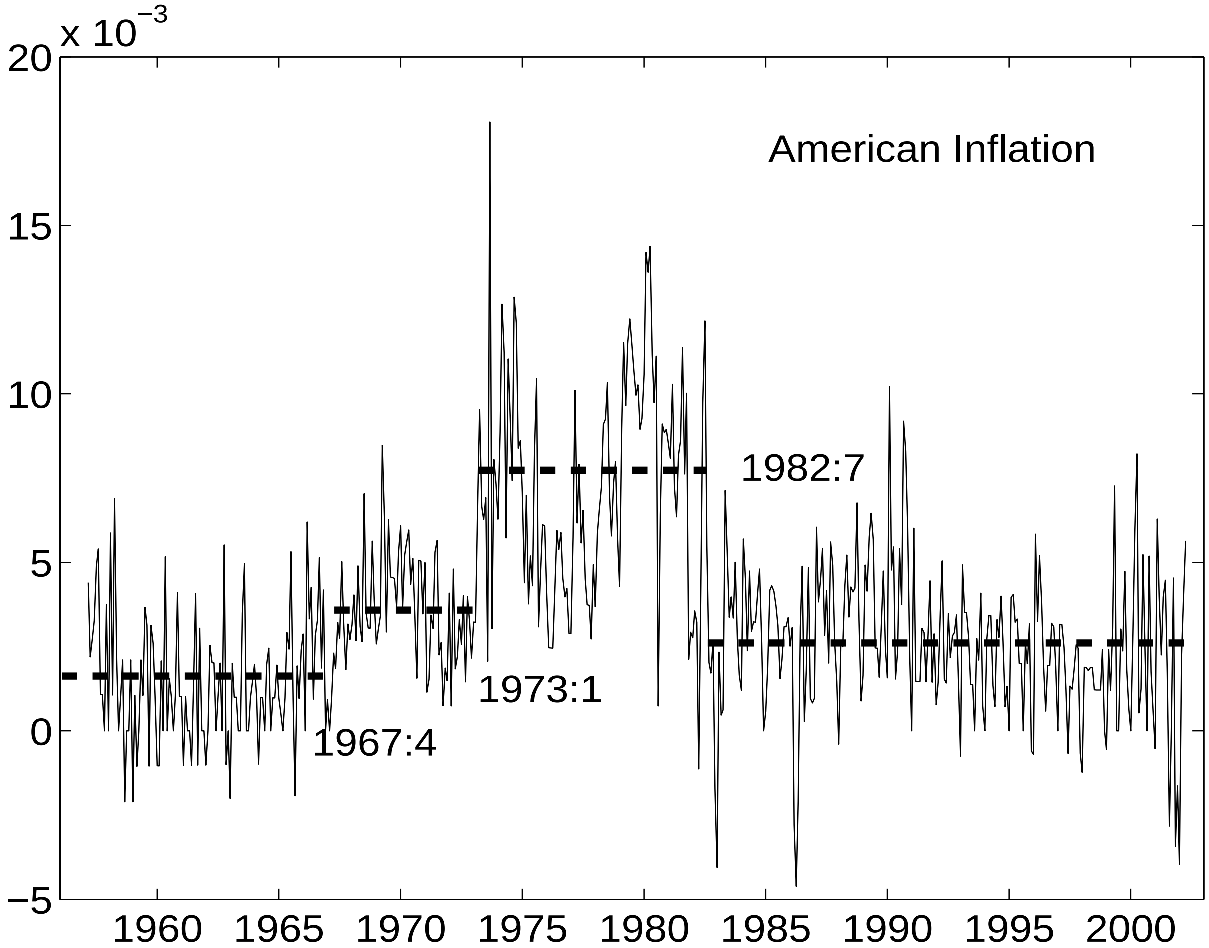
Estimated break dates (information criteria)					
France	$\tilde{T}_1$				
	1961:7				
	(61:6-61:8)				
Italy	$\tilde{T}_1$	$\tilde{T}_2$	$\tilde{T}_3$	$\tilde{T}_4$	
	1969:1	1973:7	1980:8	1985:9	
	(68:12-69:2)	(73:6-73:7)	(80:7-80:9)	(84:7-86:6)	
Germany	$\tilde{T}_1$	$\tilde{T}_2$	$\tilde{T}_3$	$\tilde{T}_4$	
	1963:9	1979:2	1989:3	1993:9	
	(62:11-66:12)	(70:6-85:2)	(86:12-89:4)	(93:8-93:10)	
U.S.A.	$\tilde{T}_1$	$\tilde{T}_2$	$\tilde{T}_3$	$\tilde{T}_4$	$\tilde{T}_5$
	1972:12	1978:8	1983:2	1991:4	1997:9
	(67:3-73:3)	(57:2-00:8)	(81:3-02:3)	(91:3-91:5)	(97:8-97:9)
Estimated break dates (sequential procedure)					
France	No break				
Italy	$\tilde{T}_1$	$\tilde{T}_2$			
	1985:6	1995:10			
	(85:5-85:7)	(95:2-02:3)			
Germany	$\tilde{T}_1$				
	1963:9				
	(57:2-65:6)				
U.S.A.	No break				

Appendix 2: Graphs of the Series

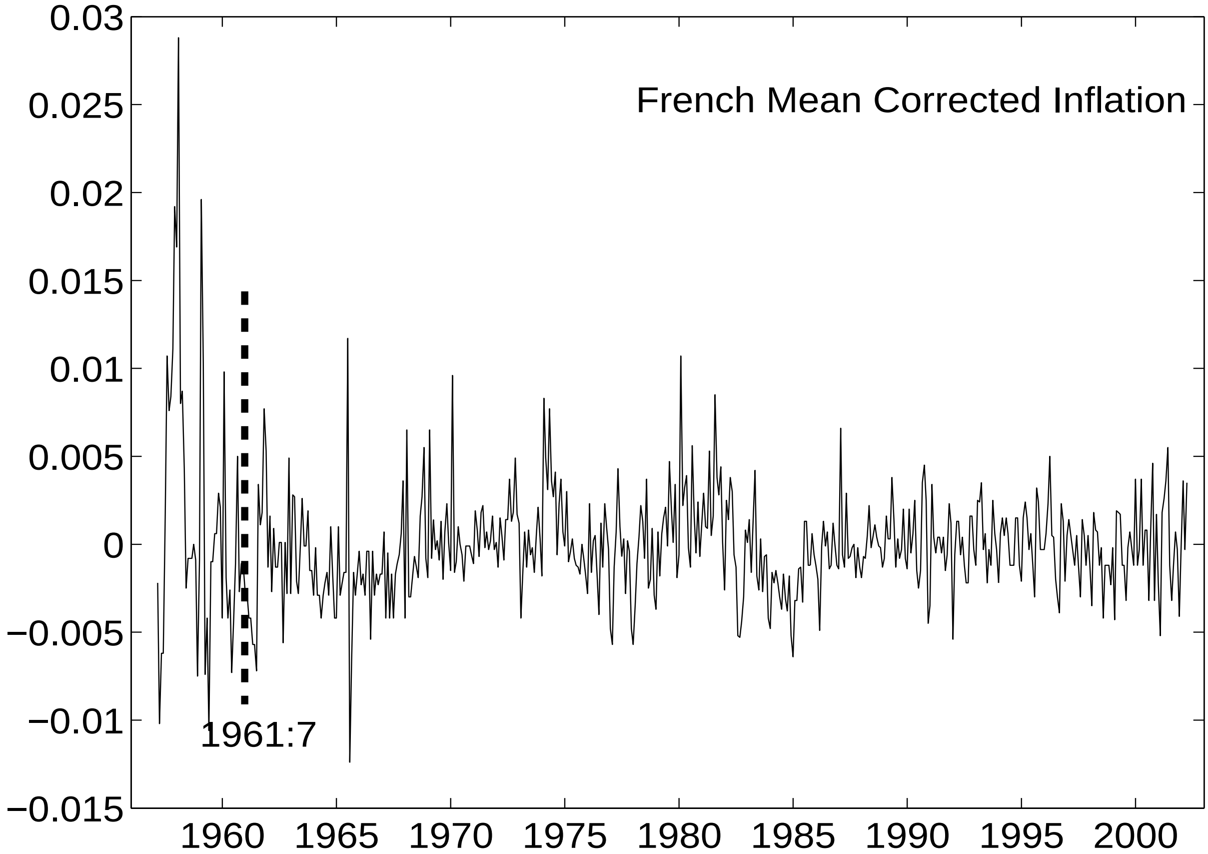


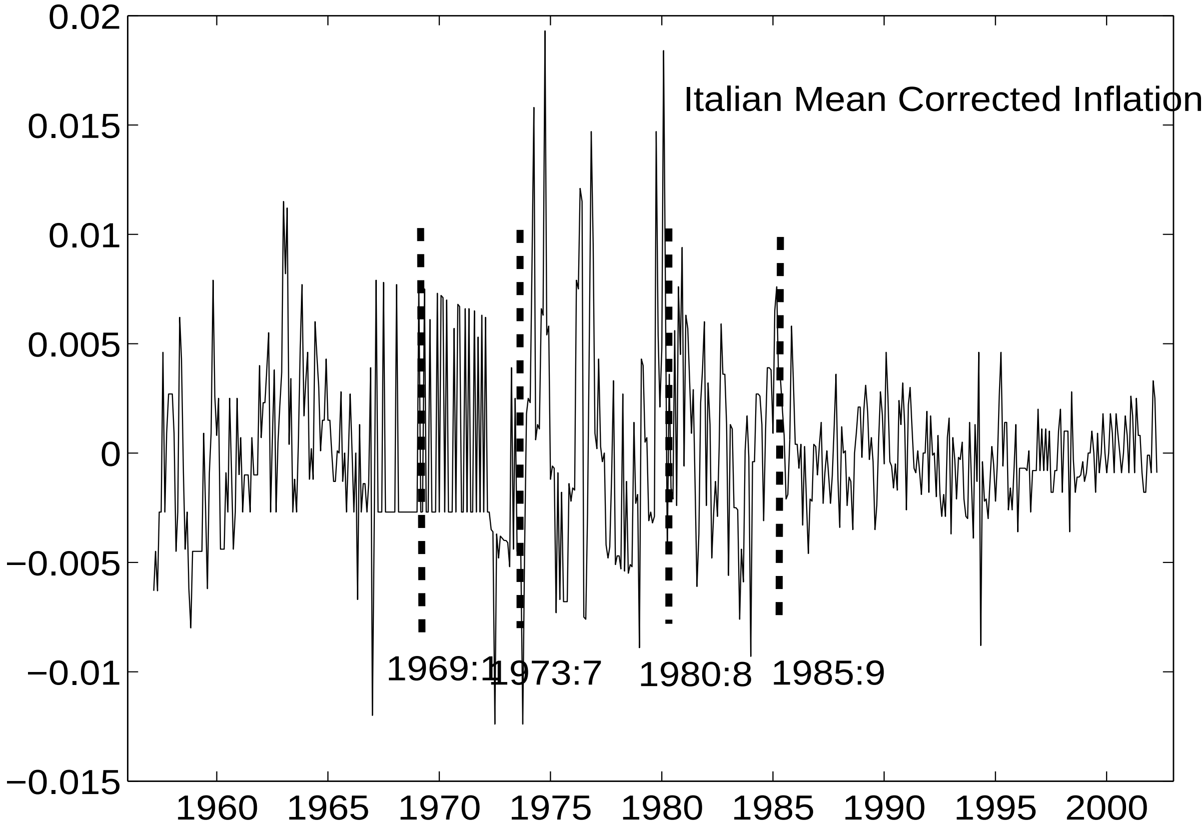


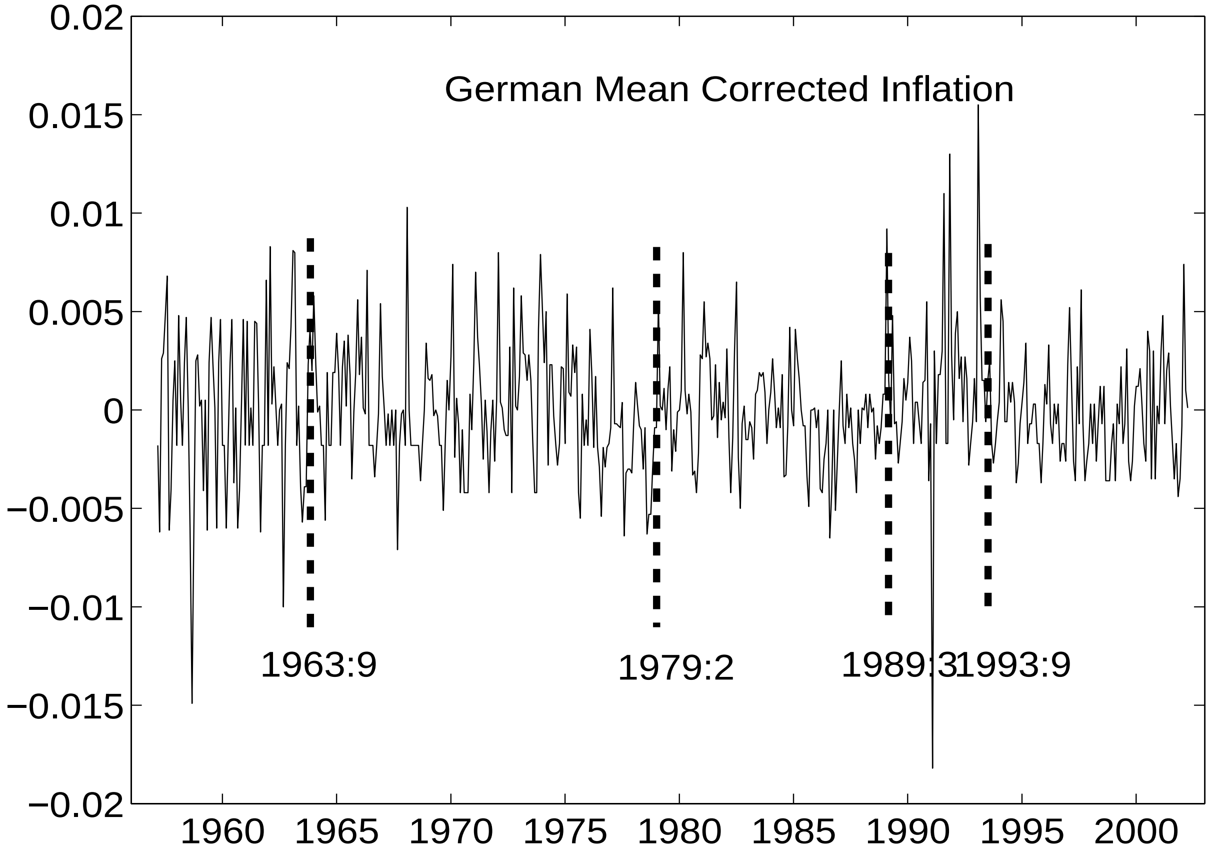

0.02
0.015
0.01
0.005
0
−0.005
−0.01
−0.015
−0.02
German Inflation
1983:1
1969:11
1960
1965
1970
1975
1980
1985
1990
1995
2000

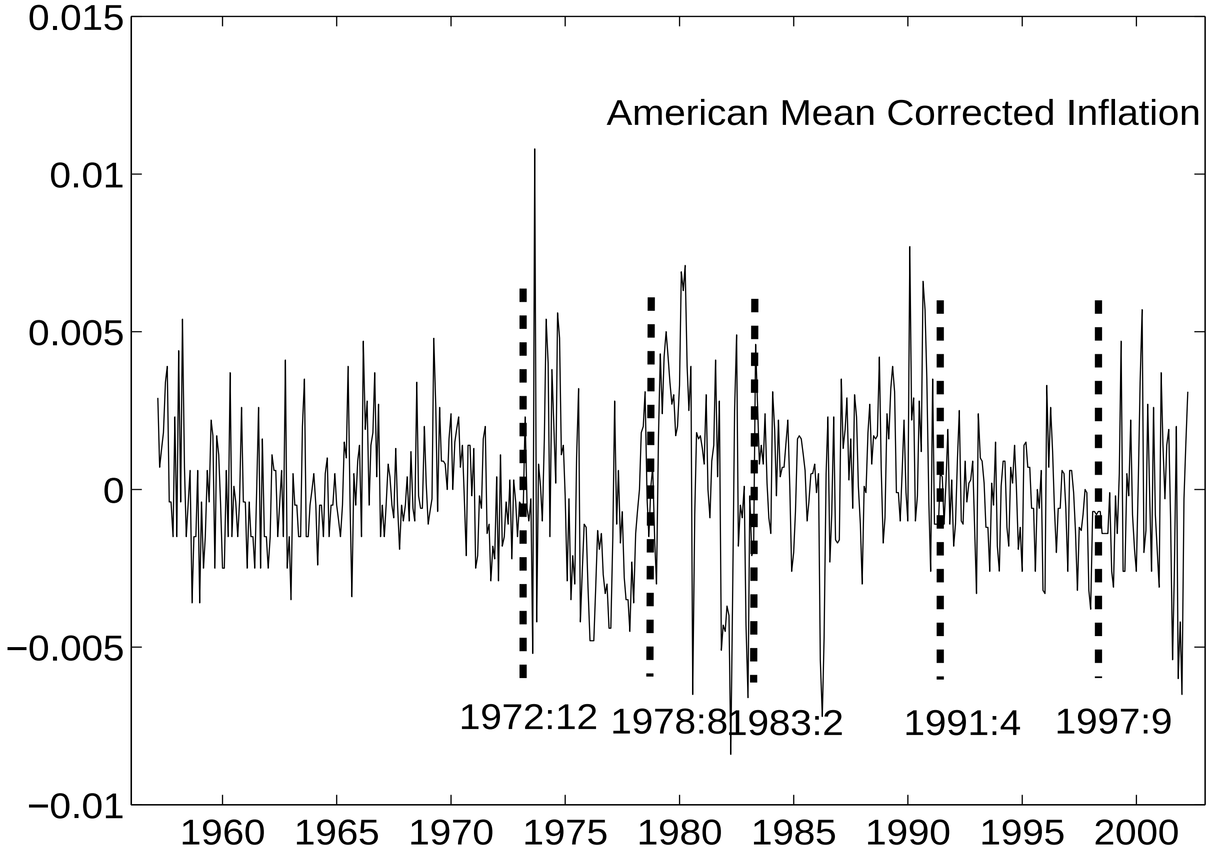

x 10^−3
20
15
10
5
0
−5
American Inflation
1982:7
1973:1
1967:4
1960
1965
1970
1975
1980
1985
1990
1995
2000

Appendix 3: Graphs of the Mean Corrected Series






German Mean Corrected Inflation
0.02
0.015
0.01
0.005
0
−0.005
−0.01
−0.015
−0.02
1963:9
1979:2
1989:3
1993:9
1960
1965
1970
1975
1980
1985
1990
1995
2000


American Mean Corrected Inflation
0.015
0.01
0.005
0
−0.005
−0.01
1972:12
1978:8
1983:2
1991:4
1997:9
1960
1965
1970
1975
1980
1985
1990
1995
2000

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